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INCOMPLETENESS OF THE FLASHING OF A SUPERSATURATED LIQUID AND SONIC EJECTION OF THE PRODUCED PHASES

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Abstract—For an adiabatic, constant flowrate and frictionless flow of a supersaturated liquid, the equations of evolution are established owing to a variable which is a flashing index. The graphical solution in the pressure volume diagram shows that the flashing is choked when the phases produced are expelled at the local sonic velocity. Comparison is made between theory and existing experimental results for hot water, the agreement is satisfactory.

1. INTRODUCTION

It is well known that the homogeneous equilibrium model of critical two-phase flow is inadequate to explain the experimental data (Zaloudek 1961; Fauske 1965; Klingebiel & Moulton 1971).

Slip and metastability have been recognized as factors able to bring theoretical estimations and experimental data in closer agreement.

The slip flow models based on thermodynamic equilibrium (Fauske 1962; Levy 1963; Moody 1965) give a better agreement between theory and experiment but the measured velocity ratios appear to be much lower than those predicted by analytical or empirical models (Henry *et al.* 1970; Klingebiel & Moulton 1971). This shows that metastability must also be taken into account, (Fauske 1965; Simon 1971; Sozzi & Sutherland 1975).

In the case of flashing, the quality x is low and visual inspection of the flow suggests a homogeneous structure in which it is difficult to imagine a high velocity ratio. In this paper, we have neglected slip and considered only the thermodynamic disequilibrium.

Our picture of the flashing phenomenon is the following: the hot liquid flows from a high pressure vessel where it is in subcooled or saturated conditions, the pressure losses (friction or else) in the pipe reduce the pressure to a value $p_0 < p_{sat}(T_E)$ and the liquid begins to boil. Figure 1 shows; (a) at the top, a sketch of the test section; (b) the point E on the pressure profile, corresponding to the onset of boiling indicated by the appearance of a void fraction (lower curve) and by the increase in the pressure gradient due to acceleration of the two-phase mixture; and (c) the thermal disequilibrium at the onset of boiling indicated by the difference $p_{sat} - p_0$ (between 0.2 and 0.3 bar in this case). Such pressure profiles have been found by Barois (1969), Reocreux (1976) and Seynhave *et al.* (1976).

In this paper, the vaporization of the metastable liquid which has reached its metastability limit (point E of figure 1) is studied, the single phase flow of the liquid upstream of point E is not considered. Then, to schematize the problem, it is supposed that, in the direction of flow, the following are met (figure 2):

An infinite source of liquid having the properties of the liquid at point E.

A thermally insulated tube where the pressure losses are considered to be due to acceleration only and where the mass flux G is set constant.

A chamber where the pressure is p.

If $p = p_0$, the liquid remains supersaturated. If $p = p_0 - \Delta p$, since the liquid has reached its metastability limit, a vaporization wave takes place where the pressure gradually decreases from p_0 down to p.

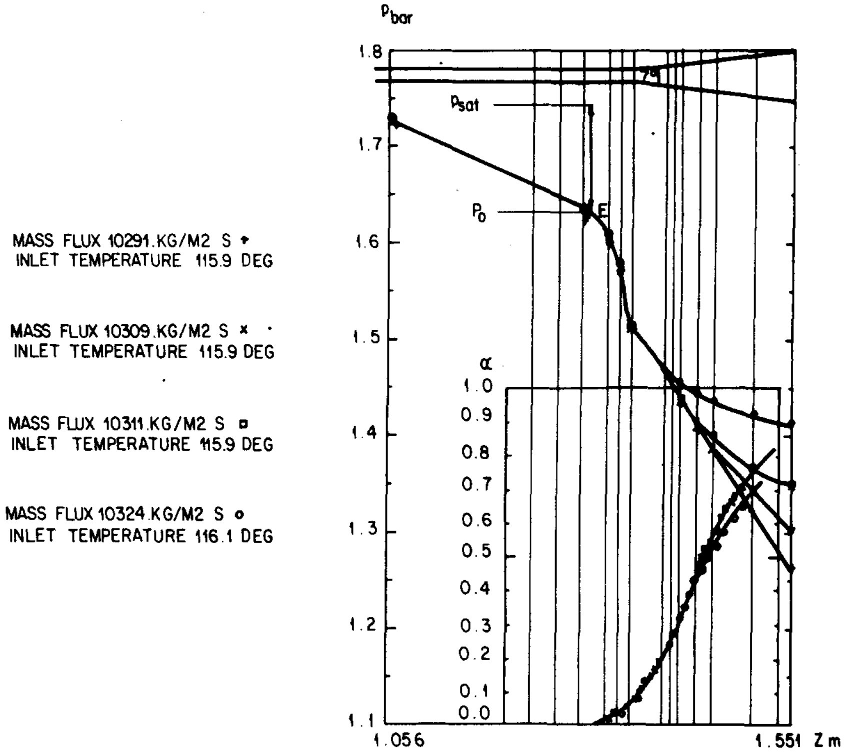

Figure 1. Piezometric line of a flashing liquid (after Reocreux 1976).

The difference between the flash model here presented and some others resides mainly in the fact that the existence of a metastable state down to p_0 is admitted. In Brauer *et al.* (1976), it is assumed on the contrary that the phase change begins immediately when the saturation conditions are met but that the return toward equilibrium is achieved at a finite velocity.

2. MATHEMATICAL MODELING

First a vaporization index y is defined which represents the fraction of the unit mass of supersaturated liquid which changes to give the mass xy of steam and $(1-x)y$ of liquid in

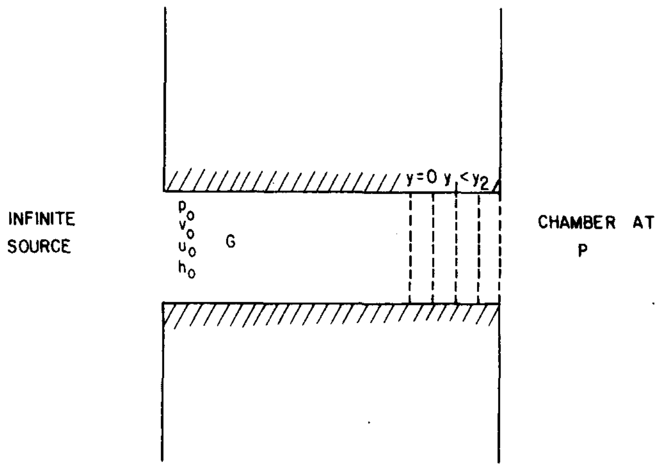

Figure 2. Scheme of the flashing conditions of the supersaturated liquid.

equilibrium at $T_{sat}(p)$. The fraction $(1-y)$ remains in a metastable state at T_E. This picture of the flow is compatible with local temperature measurements where, at a given point, it is possible to detect parts at T_E and other parts at $T_{sat}(p)$ (see Barois 1969).

The flow being adiabatic, frictionless and at constant flowrate, the vaporization index y appears as the main variable to describe the phase change.

When the vaporization wave is established in the duct, planes of increasing y are encountered where the properties of the mixture are related to these of the initial state (defined by index 0) by

$$\frac{u}{v}=\frac{u_0}{v_0}=G \qquad [1]$$

$$p_0-p=-G(u_0-u) \qquad [2]$$

$$h+\frac{u^2}{2}=h_0+\frac{u_0^2}{2}. \qquad [3]$$

Where u is the velocity of the mixture, v its specific volume and h its enthalpy. Elimination of u in [1]–[3] leads to

$$p_0-p=-G^2(v_0-v) \qquad [4]$$

$$h-h_0=\frac{1}{2}(p-p_0)(v_0+v). \qquad [5]$$

Which are completed with equations of state

$$h=(1-y)h_0+xyh_G+(1-x)yh_L \qquad [6]$$

$$v=(1-y)v_0+xyv_G+(1-x)yv_L \qquad [7]$$

where h_L and v_L correspond to h and v for the saturated liquid at p and h_G and v_G to the saturated steam under the same conditions.

Equation [5] with [6] and [7] may be written

$$v=\frac{[v_0(1-y)+yv_L](h_G-h_L)+y(h_0-h_L)(v_G-v_L)+\frac{1}{2}(p-p_0)v_0(v_G-v_L)}{h_G-h_L-\frac{1}{2}(p-p_0)(v_G-v_L)} \qquad [8]$$

which will be more simply referred to as

$$F(p,v,y)=0. \qquad [9]$$

Besides the initial conditions, [4] and [9] contain only p, v and the parameter y; it is interesting to carry on the analysis graphically in the p, v diagram.

3. GRAPHICAL SOLUTION OF THE FLASHING PROBLEM

In the (v,p)-plane, [4] represents a straight line D of slope $-G^2$ through the point $A(v_0,p_0)$,

[9] represents a family of curves F the physically reachable parts of which are shown as continuous lines in figure 3.

If the pressure p in the chamber is decreased to p_1, a rarefaction wave penetrates the tube and a vaporization wave settles down in such a way that the vaporization index in the plane at the end of the tube is y and the specific volume v. These values are found in the graph at the intersection of D with ordinate p_1. The pressure in the chamber may thus be decreased down to p_c such that D is a tangent in C to a curve F of index y_c.

A new decrease in p would induce a decrease in y which is not physically acceptable. We will see now that no other rarefaction wave below p_c can penetrate the tube.

4. SONIC EXPULSION OF THE TWO PHASE MIXTURE WHEN THE VAPORIZATION INDEX REACHES ITS MAXIMUM, y_c

The tangency of D and curve Fy_c in C is expressed by

$$\left(\frac{dp}{dv}\right)_F = \frac{p_0 - p}{v_0 - v} \tag{10}$$

Through [5] of $F(p, v, y) = 0$; we have also

$$dh - \frac{1}{2}(p - p_0)\,dv - \frac{1}{2}(v + v_0)\,dp = 0 \tag{11}$$

if T is defined as a certain temperature of the mixture, S as the entropy and if it is assumed that the thermodynamic relation $dh = T\,dS + v\,dp$ holds [11] takes the form

$$T\,dS + v\,dp - \frac{1}{2}(p - p_0)\,dv - \frac{1}{2}(v + v_0)\,dp = 0 \tag{12}$$

or

$$T\,dS - \frac{1}{2}[(p - p_0)\,dv - (v_0 - v)\,dp] = 0. \tag{13}$$

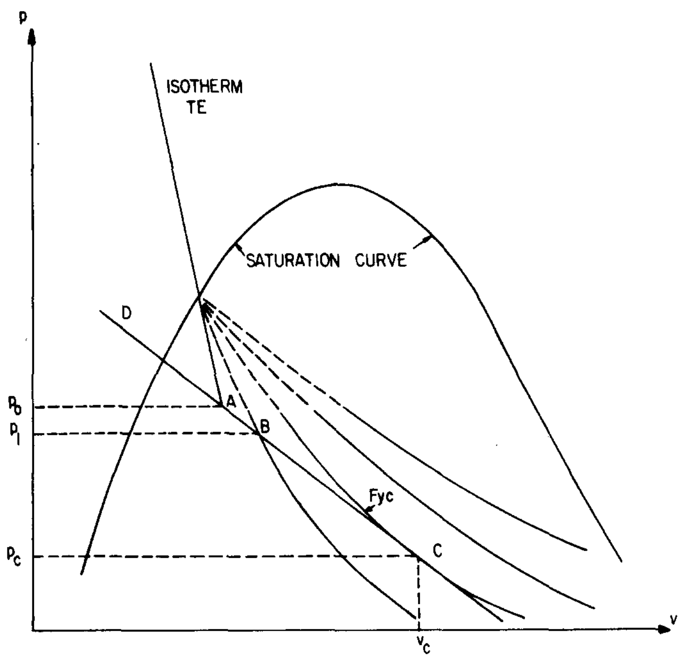

Figure 3. Graphical study of the flashing of a supersaturated liquid.

The quantity in brackets is null because of [10]. This gives at C

$$dS = 0. \tag{14}$$

From this it is concluded that the isentropic and the curve Fy_c have same tangent D at C and therefore that

$$\left(\frac{dp}{dv}\right)_F = \left(\frac{dp}{dv}\right)_S = -\frac{c^2}{v^2} = -G^2 = -\frac{u_c^2}{v_c^2} \tag{15}$$

where c is the standard isentropic sound speed.

The mixture velocity at the end of the tube equals the local sound speed. It follows that a decrease in p below p_c (which would propagate relative to the mixture at sound speed c) can no longer proceed up the tube where the vaporization index is hence limited at y_c.

5.1 *Comparison of the void fraction*

Experimental results concerning flashing of hot water are available in Reocreux (1974). Tests are made in a vertical tube 20 mm i.d., two metres long followed by a 7 deg. straight diffuser. Mass fluxes, entrance temperature and pressures along the channel are measured. The measurement of the void fraction is carried out by the X-ray absorption method.

As indicated in the introduction, the piezometric line is used to evaluate the depth of the initial disequilibrium ($p_{sat} - p_0$). Table 1 shows the data which are used for comparison. In the void fraction column, there are two figures, one for the throat and the second for a point arbitrarily fixed at 3.5 cm downstream from the throat, this will be explained later.

For each group of tests, a mean value of TE, $P_{sat}(TE)$ and p_0 has been retained. Figures 4–6 show the curves F and D for these conditions.

Though the position of the point of contact C is difficult to determine with great accuracy, it appears nevertheless that the pressure p_c is a little lower than the experimental pressure at the throat. This would suggest that the critical section is somewhat downstream of the throat. This is why two values have been given in table 1 for the void fraction. These figures are compared with the theoretical void fraction computed as follows. xy_cv_{Gc} is the volume of steam

Table 1. Data for comparison of void fractions

Pressure at the throat (bar)	Entrance Temp. (°C)	Mass flux ($kgm^{-2}s^{-1}$)	$p_{sat}(TE)$ (bar)	Pressure at the onset of flashing (bar)	Void fraction at or near the throat
1.5	116.7	4180	1.787	1.640	50–60
	116.65	6500	1.784	1.640	25–35
	116.3	8650	1.763	1.640	12–20
	115.9	10300	1.741	1.630	10–15
1.75	121.75	4360	2.098	1.938	55–60
	121.1	6500	2.055	1.925	30–40
	121	8500	2.049	1.925	20–30
	120.85	10100	2.040	1.915	15–20
2	126.1	4210	2.400	2.212	60–65
	125.4	6400	2.349	2.212	35–45
	125.2	8520	2.335	2.195	25–30
	125.1	10180	2.327	2.200	17–25

contained in the unit mass of mixture, v_c is the total volume of this unit mass; the void fraction α_c is thus

$$\alpha_c = \frac{x y_c v_{Gc}}{v_c} \qquad [16]$$

With a very good approximation, [7] becomes

$$v_c = v_0 = x y_c v_{Gc}$$

and finally we find

$$\alpha_c = \frac{v_c - v_0}{v_c} = 1 - \frac{v_0}{v_c}. \qquad [17]$$

Table 2 sums up the comparison. Taking into account the difficulty of estimating the point C in figures 4–6, and the possible experimental errors, the trend and the order of magnitude of the calculated and measured void fraction are in rather good agreement.

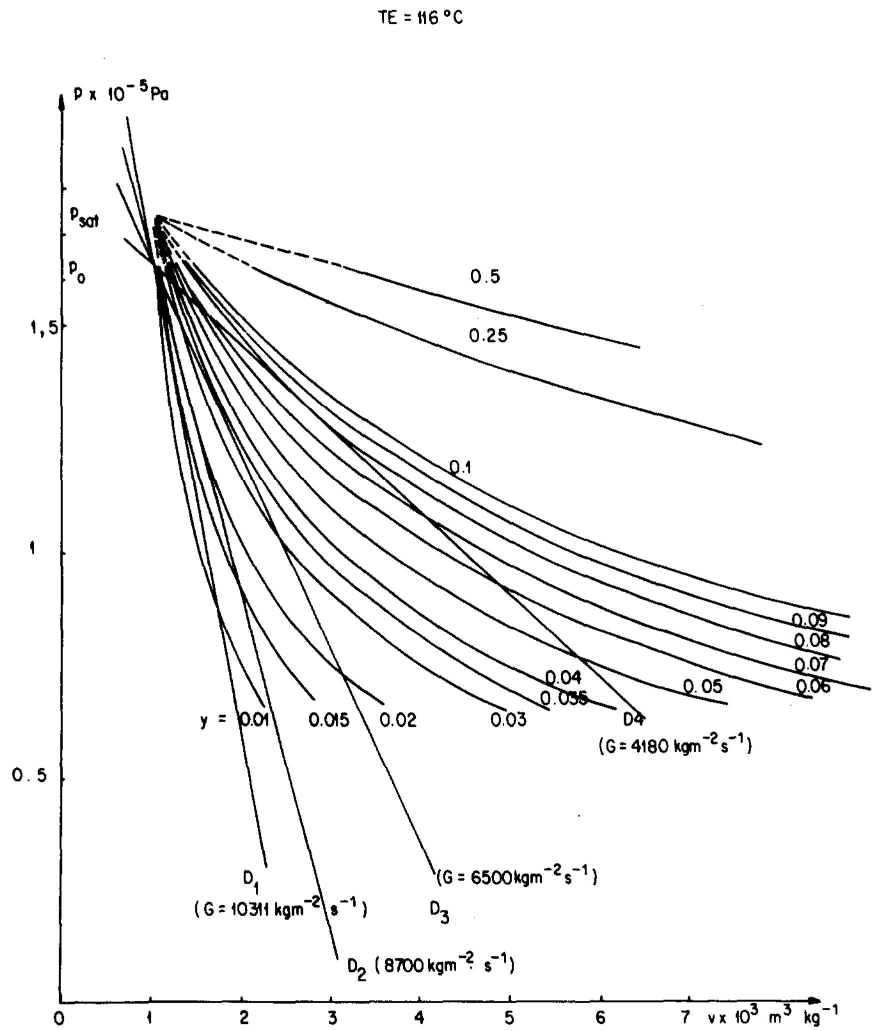

Figure 4. Determination of the critical conditions.

Table 2. Comparison of measured and calculated void fractions

Entrance temp. (°C)	p_0 (bar)	p_c read on figures 4–6 (bar)	p_c exp. (bar)	v_c read ($m^3/kg \times 10^3$)	Calculated α_c [17]	Measured α
116°	1.63	1.35	1.5	2.75	0.62	0.55–0.60
				1.75	0.40	0.25–0.35
				1.42	0.26	0.12–0.20
				1.3	0.19	0.15–0.20
121	1.925	1.6	1.75	2.8	0.62	0.55–0.60
				1.75	0.39	0.30–0.40
				1.45	0.27	0.20–0.30
				1.35	0.21	0.15–0.20
125.5	2.21	1.85	2	3	0.64	0.60–0.65
				1.85	0.42	0.35–0.45
				1.55	0.31	0.25–0.30
				1.4	0.24	0.17–0.25

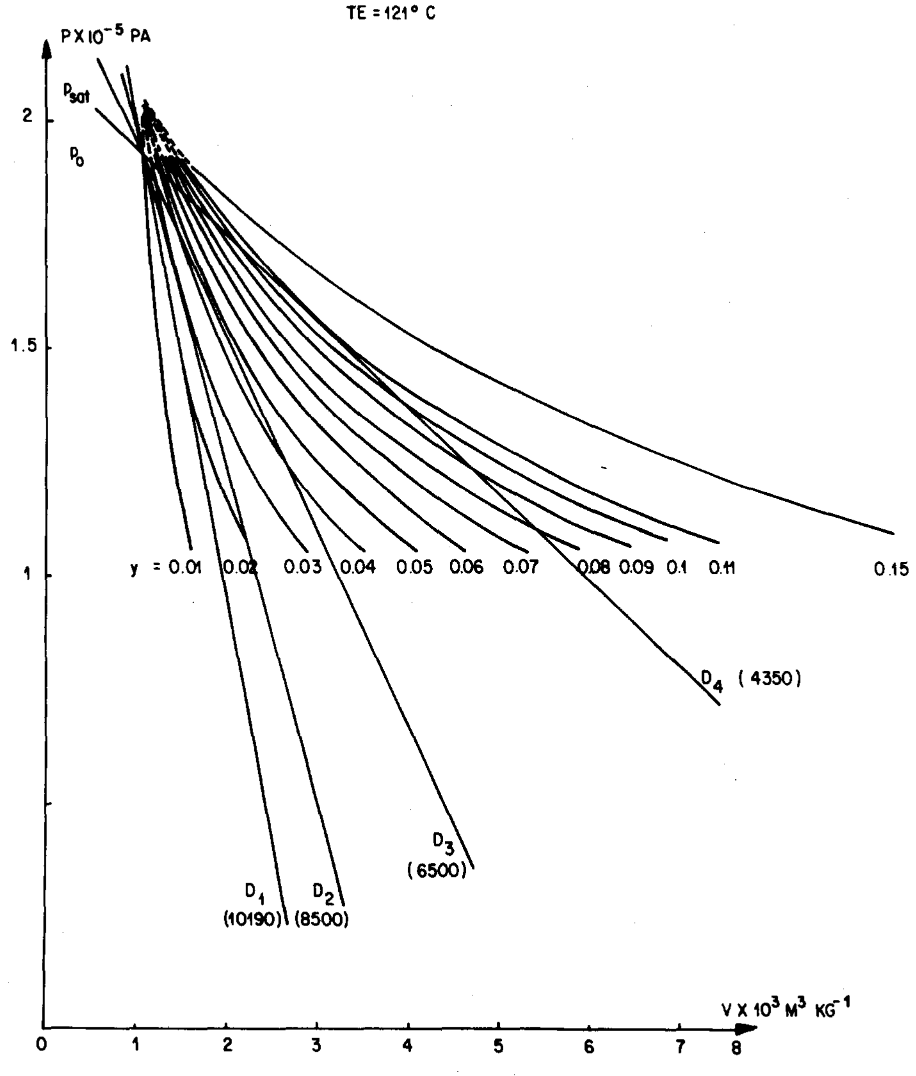

Figure 5. Determination of the critical conditions. $TE = 121°C$.

5.2 The pressure loss by acceleration does not depend on the mass flux at constant inlet temperature

At least at low pressure, this has been found experimentally by Reocreux (1976) and Seynhaeve *et al.* (1976). It is a consequence of the fact that inside a group of tests at the same inlet temperature and the same pressure at the throat (see table 1), the value of the pressure p_0 which involves the onset of boiling is nearly constant.

For given values of p_0, figures 4–6 show conversely that p_c is almost a constant for the same inlet temperature. Consequently, the difference $p_0 - p_c$ is a constant. The demonstration is made more accurate by studying the locus of point C in the (v, p)-plane. This locus is determined by

$$v_c = f(p_c, y_c) \tag{18}$$

$$v_c - v_0 = -\frac{1}{G^2}(p_c - p_0) \tag{19}$$

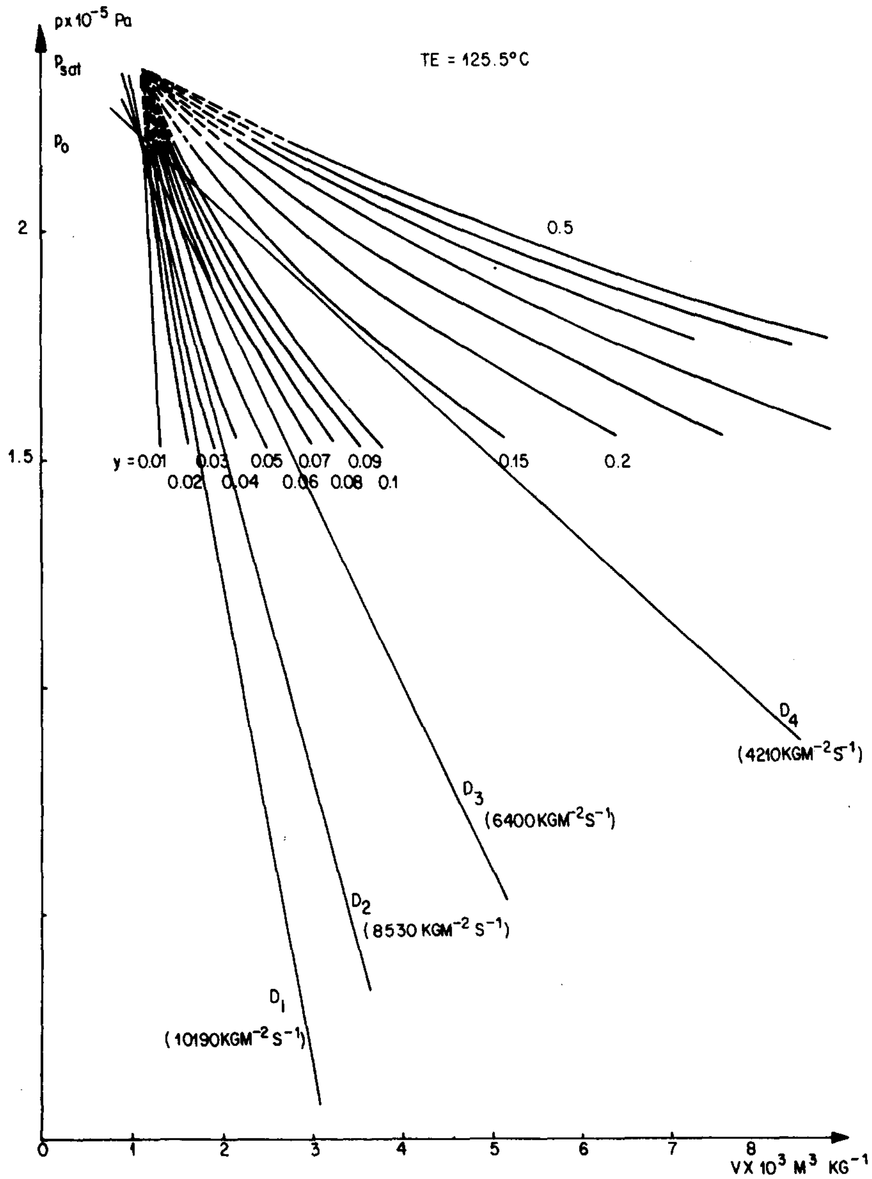

Figure 6. Determination of the critical conditions. $TE = 125.5$°C.

$$\left(\frac{\mathrm{d}}{\mathrm{d}p} f\right)_{\mathrm{at}\, C} = -\frac{1}{G^2}. \qquad [20]$$

The function f of [18] is

$$v \equiv f \equiv \frac{v_0(h_G - h_L) + \frac{1}{2}(p - p_0)v_0(v_G - v_L)}{h_G - h_L - \frac{1}{2}(p - p_0)(v_G - v_L)} + y\,\frac{(v_L - v_0)(h_G - h_L) + (h_0 - h_L)(v_G - v_L)}{h_G - h_L - \frac{1}{2}(p - p_0)(v_G - v_L)} \qquad [21]$$

with an accuracy better than 5%, it can be shown that†

$$v \simeq v_0 + y\,\frac{h_0 - h_L}{h_G - h_L} v_G \qquad [22]$$

and

$$\frac{\mathrm{d}v}{\mathrm{d}p} \simeq y\,\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{h_0 - h_L}{h_G - h_L} v_G\right). \qquad [23]$$

By eliminating G and y_c in [18]–[20], we have

$$(v_c - v_0)^{-1}\left[1 - \frac{f_1(p_c)}{(p_c - p_0)f_1'(p_c)}\right] \simeq 0 \qquad [24]$$

where f_1 is a function of p alone set for

$$\frac{h_0 - h_L}{h_G - h_L} v_G.$$

Since $v_c \neq v_0$, the quantity in brackets is nearly 0 which implies

$$p_0 \simeq g(p_c) \quad \text{or} \quad p_c \simeq g^{-1}(p_0). \qquad [25]$$

The approximate constancy of p_c and of $(p_0 - p_c)$ is thus established when the mass flux varies at constant inlet temperature.

5.3 *Form of the critical relationship*

By critical relationship we mean a relationship between the mass flux G and the conditions in the critical section. With [19] and the approximation

$$v_c - v_0 = x y_c v_{Gc}. \qquad [26]$$

We have

$$G^2 = \frac{p_0 - p_c}{v_{G_c}} \times \frac{1}{X_c} \qquad [27]$$

†At low pressure (1), $v_0 \simeq v_L \ll v_G$; (2), the enthalpy term and the compression term differ by nearly two orders of magnitude.

where $X_c = xy_c$ is the effective quality in the critical section. If X_c is expressed *in per cent*, we find

$$G = 10\sqrt{\left(\frac{p_0 - p_c}{v_{G_c}}\right)} \times \frac{1}{\sqrt{X_c}}. \qquad [28]$$

The values of X_c computed by Reocreux (1974) from his experiments with the assumption of a velocity ratio equal to 1 are in table 3.

These values are plotted on figure 7 where, for comparison, the theoretical lines representing [28] have been drawn with the following data (table 4)

It may be concluded that at least for very low qualities [28] is a good critical relationship.

Table 3. Calculated qualities from Reocreux's (1974) experiments (velocity ratio = 1)

TE (°C)	*G* ($kgm^{-2}\,s^{-1}$)	X_c (%)	p_c (bar)
116.7	4193	0.099	1.5
116.7	6526	0.026	
116.3	8709	0.010	
115.9	10291	0.005	
121.9	4383	0.130	1.75
121.3	6519	0.043	
121.1	8474	0.023	
120.8	10111	0.017	
126.2	4218	0.188	2
125.5	6410	0.065	
125.1	8520	0.034	
125.0	10176	0.023	

Table 4. Data for drawing theoretical lines of figure 7

p_0 (bar)	p_c (bar)	v_{G_c} ($m^3\,kg^{-1}$)	$10\sqrt{(p_0 - p_c/v_{v_c})}$ ($kgm^{-2}\,s^{-1}$)
1.63	1.5	1.159	1.059×10^3
1.925	1.75	1.004	1.32×10^3
2.21	2	0.8854	1.54×10^3

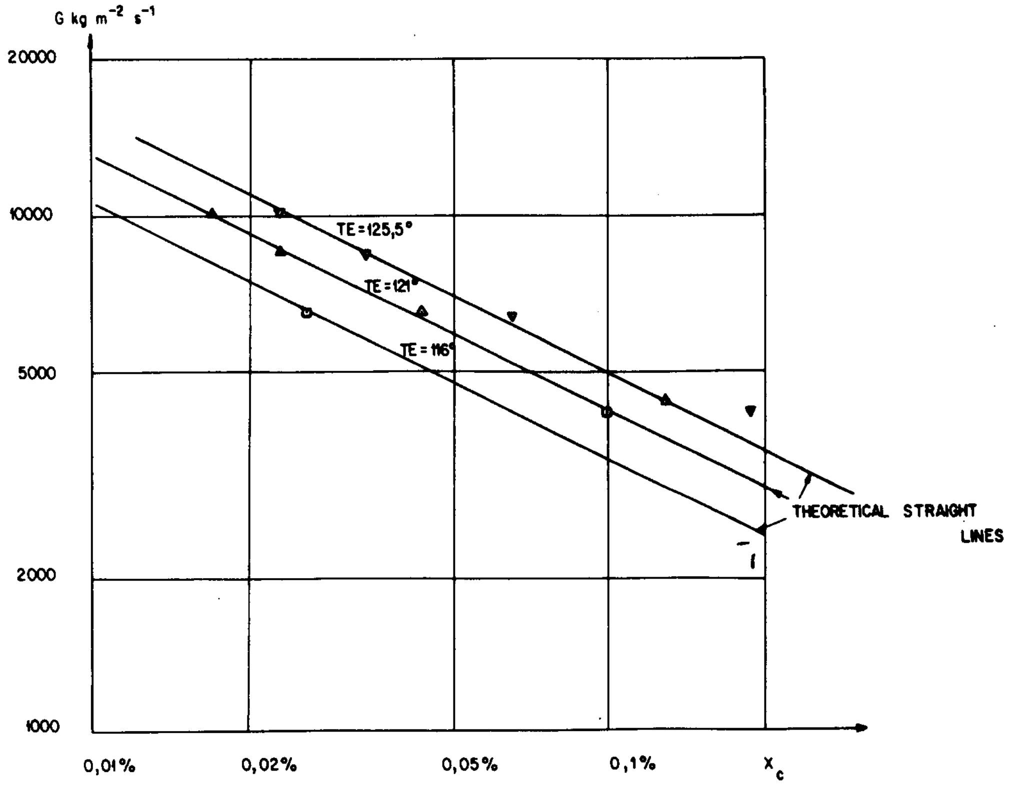

Figure 7. Comparison of the theoretical result $G \sim X_c^{-1/2}$ with experience.

6. CONCLUSIONS

A model for the flashing of a supersaturated liquid has been developed. It follows from this model that vaporization is necessary incomplete leaving about 90% of the liquid in supersaturated state. When vaporization is choked, the vaporization products are expelled at the local sonic velocity.

This model has been applied to the flashing of hot water at low pressure ($p \leq 2$ bar). There is a good agreement between the measured and calculated void fraction. The constancy of the pressure loss by acceleration during the vaporization process is deduced from the model for the conditions of the tests. A simple form of a relationship between the mass flux, the pressure at the onset of boiling p_0 and the effective critical quality is verified.

The pressure p_0 plays a central role in the analysis; it is an important property of the water and emphasizes the need to know more about nucleation and the triggering of boiling.

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